

TEMPERATURE MEASUREMENT AND CONTROL

C Muirhead

University of Birmingham

TEMPERATURE: GENERAL CONSIDERATIONS

Absolute temperature is defined in thermodynamics by:

$$dS = \frac{dQ_{rev}}{T} : \text{entropy / reversible heat relationship}$$

or in statistical mechanics by:

$$\frac{1}{T} = k \left(\frac{d \ln \Omega}{dE} \right)_V : \text{change of no. of microstates with energy.}$$

Either of these definitions require one fixed point (the other is absolute zero) to give the size of the temperature unit, the Kelvin. This defined fixed point is the triple point of water $\approx 273.16\text{K}$ ($\sim 0.01^\circ\text{C}$). To make absolute temperature measurements one must find a sufficiently well understood system (e.g. a gas, an electrical resistor of fixed value, a paramagnetic salt) and measure its properties (e.g. pressure, speed of sound, electrical noise, magnetic susceptibility). However, these measurements are too cumbersome for most practical purposes, so by international agreement, “practical temperature scales” have been established. These give the best known absolute temperatures of various subsidiary fixed points and means for interpolating between them. The fixed points are the temperatures at which various phase transitions occur (e.g. T_c of lead, triple point of O_2 , etc.). Any properly calibrated thermometer will have its calibration traceable back to a practical scale and hence to absolute temperature. However, every few years the progress in absolute thermometry shows up the inaccuracy of earlier scales and a new practical temperature scale is established. The most recent ones are (number = year) IPTS-68, EPT-76, ITS-90. For highest accuracy, you need to know to which scale a thermometer was calibrated. For instance, commonly-used vapour pressure tables for He^4 were established in 1958, and differ from T_{90} (or T_{76}) by $\sim 7\text{mK}$ at 4K . (Fig. 1 shows comparisons between T_{90} and earlier scales).

For temperatures above 1K it is unlikely that you will be calibrating your own thermometers. In this respect, much of the early part of the Ch.4 of White is irrelevant: you do not need to know about gas thermometry, but instead about what is the most suitable resistance or diode thermometer, which you will then buy in pre-calibrated. If you need to check this calibration, the easiest technique is to compare with another calibrated thermometer, or to use various fixed points. For temperatures below 7.3K , the NBS supplies a set of superconductors of known T_c mounted with coils for the inductive determination of T_c s (device SRM767).

The normal boiling point of He^4 can also be used to give a fixed point with an accuracy of a few mK. Boiling liquid N_2 can be relied on to a fraction of a K, so long

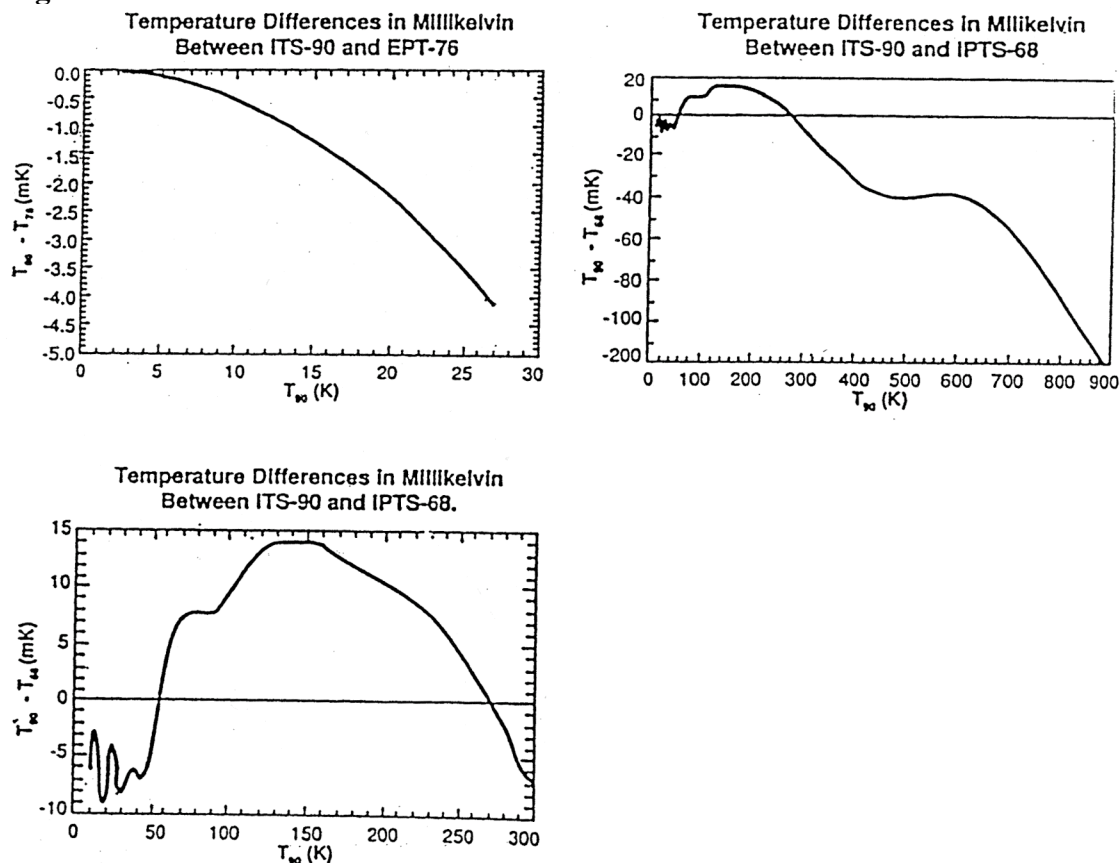
as it is fresh and not contaminated with oxygen condensed from the air. LHe, LN₂ and melting ice are probably sufficient to fix a temperature scale for most HT_c work.

INTERNATIONAL TEMPERATURE SCALE OF 1990

Effective January 1, 1990, the ITS-90 is the official international temperature scale. This scale was developed to overcome certain deficiencies that existed with IPTS-68 and EPT-76. In particular, temperatures corresponding to ITS-90 are in better agreement with the true thermodynamic temperature than previous scales. This scale change must be taken into account for standards applications and other requirements for precision thermometry. All Lake Shore sensors are now calibrated on ITS-90.

The figures below indicate the difference between temperatures on ITS-90 (T_{90}) and temperatures on EPT-76 (T_{76}) and IPTS-68 (T_{68}). The graphs represent a smoothed curve to the difference tables given by B.W. Mangum in his note "A New Temperature Scale, The International Temperature Scale of 1990, Is Adopted", available from NIST in Gaithersburg, Maryland. Only temperature ranges relevant to Lake Shore sensors are represented.

Figure 1.



SMOOTH INTERPOLATIONS

Highest accuracy calibrations will be required if you are measuring things like dp/dT thermal conductivity or heat capacity. The values of these depend on the *slope* of

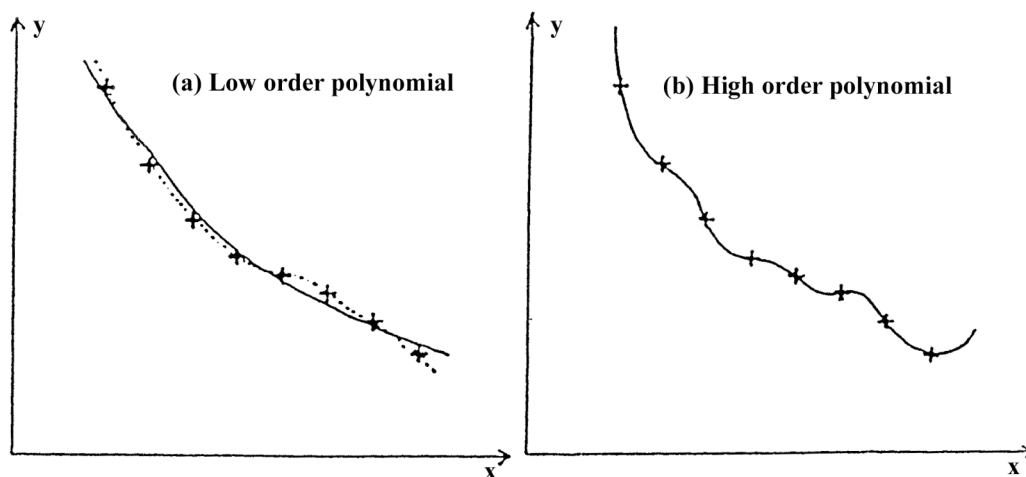
your temperature calibration and if that is not smooth neither will the results be. Indeed, you may need to use results from a featureless sample to smooth out noise in a commercially supplied calibration.

Interpolating or fitting calibration data can be a problem in a high accuracy work: there are no simple physical expressions for, for example, the resistance R of a germanium resistance thermometer versus temperature, so polynomial expressions with plenty of mathematical flexibility but no physical justification are often used e.g.:

$$\ln R = \sum_n a_n (\ln T)^n$$

It is up to the experimenter to decide how many terms to use in the polynomial. Unfortunately, low order polynomials are unable to fit the data over a wide range, but high order ones develop wiggles to try to fit the noise and also diverge away immediately outside the fitting range (figure 2). Either of these have a deleterious effect on the slope of the calibration.

Figure 2. Polynomial Fits



I prefer to use a ‘cubic spline’ interpolation fitting. Suppose you have a certain number of points on a calibration graph which you wish to interpolate. The cubic spline is a function which passes through all the points (usually known as ‘knots’) with a smooth first derivative, a second derivative which varies linearly from knot to knot, but discontinuous third derivative. It thus varies locally as a low-order polynomial, but one which adjusts its coefficients as one moves along the graph. The name comes from the fact that it is the shape which would be taken up by a perfectly elastic thin strip (spline) bent to pass through the knots. If the number of knots equals the number of data points it is an interpolation; if the number of knots is smaller than the number of data points, the knot y-values can be varied to give a least squares fitting. In either case, the slopes at the ends of the range have to be fixed. For further details see books on numerical techniques.

In any case, an interpolation or fitting will be most successful, with fewest fitting parameters, if the curve has as little curvature as possible. For germanium resistors and carbon resistors at low temperatures, I have found that

$$\ln R = A + B / T^\beta$$

with $\beta < 1$, is not too far from a straight line. Once you have fixed on a suitable value for β , fitting a curve for $\ln R$ against $T^{-\beta}$ will therefore be more successful than fitting R against T .

TYPES OF THERMOMETER

Thermometers may be put to two uses:

- i. A “primary” calibrated thermometer to provide a stable link to an absolute temperature scale, with greatest or lesser accuracy, convenience, response time or magnetic field dependence.
- ii. A “secondary” thermometer which may have a smaller size, heat capacity, response time, magnetic field sensitivity, cost or reproducibility than i. It will probably have to be calibrated using a primary thermometer, perhaps every run. There is a continuous spectrum of thermometer properties, so for rough work, a thermometer which some would say is of type ii. be used as if it were of type i.

The highest accuracy primary thermometers in the 1-300 K range are resistance thermometers:

a) Germanium Resistance Thermometers

R falls rapidly with increasing T. Very high sensitivity at low T, but falls off at high T. Suitable models can be used down to 50mK, and others up to 30 (or even 100) K. R very badly affected by magnetic fields at low T.

b) Platinum Resistance Thermometers

Useful accuracy and sensitivity above 30K. Slow response. Affected by magnetic fields at low temps. Used in their appropriate temp ranges, *a)* and *b)* will together provide the most accurate temp scale for 1-300 K in zero magnetic field.

c) Carbon-Glass Resistors

Somewhat less accurate than *a)*, but very much less magnetic field sensitive. Useful over a similar temperature range with similar R vs T characteristics.

d) Rhodium-Iron alloy resistors

Iron as magnetic impurity in Rh gives this a useful and not too variable dR/dT over the whole range 1-300 K (a pure metal or one with non-magnetic impurities would flatten out at low T). However, as a consequence, R is B-field sensitive at low T. The combination *c)* + *d)* gives reasonable accuracy and if sensors used in appropriate ranges, reasonable field insensitivity up to a few Tesla.

e) Si and GaAlAs diodes

Temp indication is voltage drop across forward-biased diode at const current of $10\mu\text{A}$. Big signal ($\sim\text{V}$ at low T). Accuracy good enough for high T_c 's. Too much power dissipation for accurate use in He temp range. Can be diodes conforming to standard V-T curve with given accuracy. Cheapest from Institute of Cryogenics, Southampton. Advantages: small size, large signal, quick response-time; but Si diodes very field-dependent at low T, also there is a smooth-calibration-destroying kink in V-T curve at approx 20K; diodes can be destroyed by heating too much above room T. (Economy note: red LED's can be operated as diode temperature indicators).

f) Thermocouples

Can be made very small size with very rapid response time; automatic measurement of temperature differences using two junctions. Insensitive below 4K and sensitivity at low temperatures only obtained at expense of magnetic field dependence caused by using alloys with magnetic impurities as elements. Temp gradients in a lead of a single material can give spurious emfs arising from non-uniformity, strains etc. If using at low T, reduce errors by having reference junction at low T, not room T. T couples are not for accurate temperature measurement but are useful in special cases.

g) Carbon Resistors

These behave like semiconductors, though graphite is a metal so probably temp-dependence is a consequence of conduction in a granular material. Not very reproducible, so ought to be used as secondary thermometers. "Allen Bradley" for 1-20K has low magnetoresistance, similar to carbon-glass. "Speer": suitable below 1K (A-B dR/dT too large here). Both can be ground to thin wafers to give rapid response.

h) Capacitance Thermometers

Rely on change of dielectric const. of a ferroelectric with T. Only thermometer apparently unaffected by B-field up to 20 T. However suffers drifts after temp changes and requires capacitance bridge to measure. Only for use in establishing calibrations in high fields. This may perhaps be better done by situating ordinary thermometers outside the field.

Overview

See tables I and II. Get a copy of Lake Shore temperature sensor guide (booklet) or their wall chart: both available from Cryophysics Ltd, Witney, Oxford OX8 6YD.

PRACTICAL CONSIDERATIONS

a) Temperature Uniformity

The most important thing for a thermometer is to be sure that it is at the temperature you wish to measure! Suppose that in a vacuum can you have a copper block raised to 10 K by a heater, with heat flow down a brass rod to surroundings at 4 K (fig. 3). Arrangement 3 a) relies on the thermal conductivity of copper to reduce the difference between thermometer and sample temperatures

which arises because of the heat flow. Always use something like arrangement 3 b), which is far preferable, since the thermometer and sample are at a “thermal dead end” into which no heat flow occurs when in a steady state.

Actually, this isn't quite true, because the thermometer must be connected to the outside by its electrical leads, which can be a source of heat or cool conducted directly to the thermometer. This is of some importance, since many thermometers are better thermally connected to their electrical leads than to their casings. Thus the leads must be well “thermally grounded” to the thermometer block, and the external electrical connections must have low thermal conductivity – i.e. be made of resistance wire (or below 7K, Nb-Ti s/c wire, with the Cu cladding removed with HNO₃ except at the ends where it is to be soldered). A suitable way of mounting a thermometer is shown in fig. 4. If the leads have come directly from room T in a vacuum they should be thermally grounded on the way down and not just at the thermometer.

Figure 3. Assuring temperature uniformity

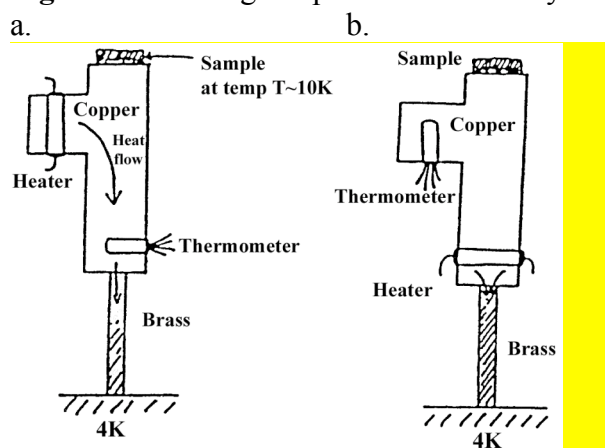
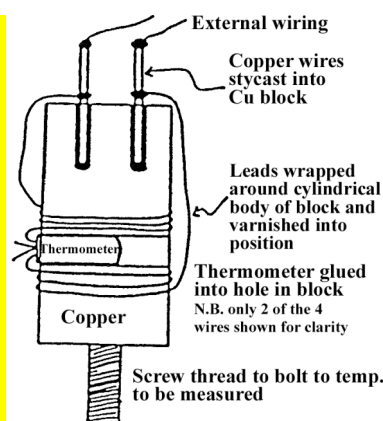


Figure 4. Thermometer mounting



Near 100K, thermal radiation exchange with surroundings is important; it is also more common to use heat exchange gas. In either case, a thermometer and its immediate leads should be within a copper can to ensure an isothermal region. (The copper can need not be thick: at this temp, $K(\text{Cu}) = 400 \text{ Wm}^{-1}\text{K}^{-1}$: approx 6000x $K(\text{He gas})$. The latter is independent of pressure for p above 0.1 torr. At 100K thermal radiation density is $< 1 \text{ mWcm}^{-2}$).

b) Self-Heating

I^2R heating may mean that the sensing element of a thermometer is not at the temperature of its surroundings. This is normally more of a problem at low temps since K is usually lower at low T and ΔT is also a larger fraction of T. For NTC resistance thermometers like carbon, Ge in which R rises rapidly at low T, the current must be reduced as T is reduced. In resistors, self-heating may easily be checked by doubling the current (4x the power) and seeing if the resistance changes noticeably. Calibrated thermometers come with stated maximum currents to avoid this effect: e.g. 1 μA : 1-3K; 10 μA : 3-30K; 100 μA : 30-300K. Such constant current values make the resistance easy to read directly as a voltage, but whenever the current is changed, the power is changed by a factor of 100. Clearly, if 10 μA does not cause noticeable self-heating just above 3K then

1 μ A is far smaller than necessary just below 3K. Better sensitivity is therefore obtained by using a variable current. Indeed, a constant voltage of approx 10mV across a Ge resistance thermometer – leading to self-heating falling off as V^2/R at low temps – will probably give negligible self-heating at all temps.

Diode thermometers are operated at const current and the voltage across them rises at low temps. Thus it is difficult to test for self heating, which can be a problem in the He temp range unless the diode is carefully thermally grounded.

c) Magnetic Field Effects

A Ge resistance thermometer can double in resistance in a field of a few Tesla at low temperatures. Therefore think about this when designing apparatus for use in a field. Can you run a long copper rod to a low field or shaded region – and there have a primary thermometer or a secondary thermometer to transfer the calibration to high field? See table II (or “Cryogenics” Nov 1977) for indication of size of effects in various thermometers.

d) Attachment of Thermometers to Apparatus

Best thermal contact occurs with clean-metal to clean-metal electrical contact. This gives smallest temp error, and quickest response time. Thermal conductance of contact proportional to pressure (because area of actual contact proportional to pressure). Grease may lubricate contact and keep it clean; Cu-powder loaded grease may hold surfaces apart! Once again, don’t forget to thermally ground the leads! See White Ch V for more details.

TEMPERATURE CONTROL

a) Thermal Time-Constant of Sample Region

Suppose you wished to hold a block of copper at 100K with cooling provided by a heat link to a He bath at 4K which contained say a SQUID or superconducting magnet, the copper represents a thermal capacity $C \text{ JK}^{-1}$ and the link a thermal resistance $R \text{ KW}^{-1}$ (i.e. power $P = \Delta T/R$ with $1/R = (\text{Avg Thermal Cond}).\text{Area}/\text{length}$). The product of these two: $\tau = CR$ is the thermal time constant (in secs) for the copper to cool. Suppose we have 7 cc of Cu (i.e. 1 mole: $C=16 \text{ JK}^{-1}$ at 100K= $\theta_{\text{Debye}}/3$) and we require $\tau = 5$ minutes. This gives $CR=300\text{s}$ i.e. $R=20 \text{ K/W}$ and the heat flow P down the thermal link = 5W (from $T=PR$ analogous to $V=IR$). This power will boil off 7ltr of liquid He per hour! This is far too much. The only ways out of this are to:

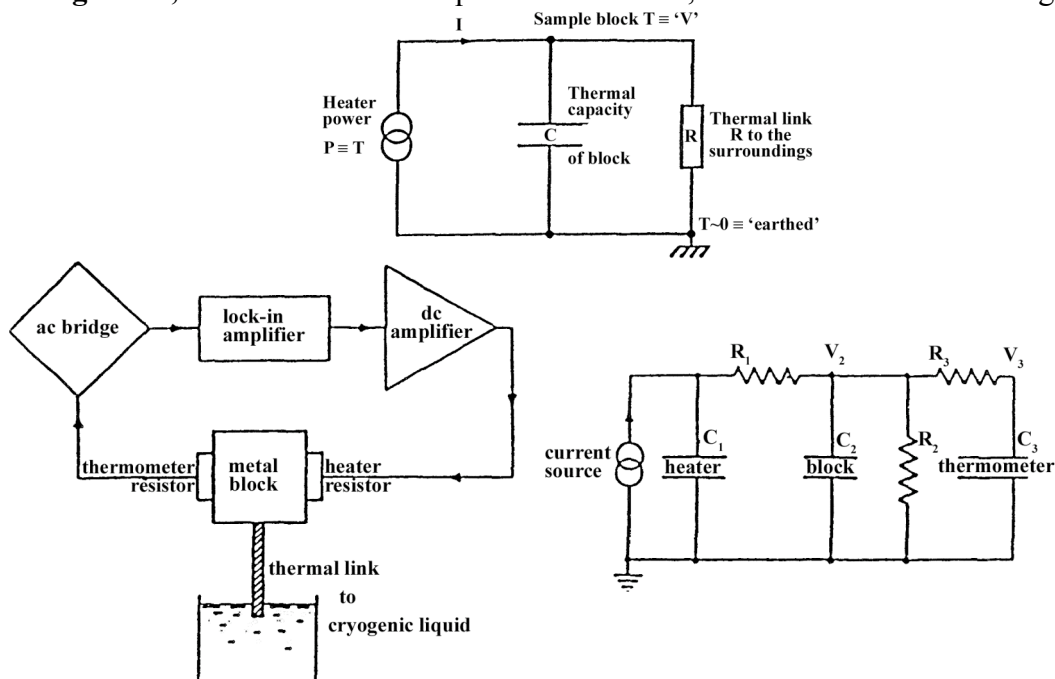
- i. Reduce mass of copper which is to have its temp controlled.
- ii. Accept much longer time constants – entailing longer cooldown times at beginning of experiment and slower temperature changes during expt.
- iii. Dump heat to higher temperatures, e.g. to He gas (as happens to a certain extent in a continuous-flow cryostat) or to LN_2 . This example shows how important it is to consider temperature-control matters at an early stage in design.

Note that to heat the block at a similar rate to that at which it cools requires a heater power twice that which will maintain the steady state temp. The time-response of the system may be conveniently imagined and modelled by the CR circuit shown in fig.5.

b) C-R Modelling of Thermal Circuit for Temperature Control

If we now try to control the temperature, we need to model the system more carefully. The physical form of the system is shown in Fig.6, and the CR analogue of the thermal part of this closed control-loop is shown in Fig.7. Note that these CR combinations, which can be roughly described as three thermal time constants – those of heater, sample and thermometer lead to phase lags in any thermal signal in the system. This can lead to unstable operation if the overall gain is large enough for the whole feedback system to act as a phase-shift oscillator.

Figures 5, 6 and 7. Electrical equivalent of heater, block and cold surroundings



c) Proportional Integral and Differential (PID) Control

Let us suppose that the bridge in fig.6 is set to balance at the desired value of thermometer resistance. If the actual thermometer resistance corresponds to a lower temperature, and the sign of the feedback is correct, the out-of-balance signal of the bridge will lead to a heater power tending to raise the block temperature. However the block will never stabilise exactly at the desired temperature because the heater signal is proportional to the out-of-balance signal from the bridge, which therefore has to be nonzero. The error can be reduced by increasing the gain, but there is a limit to this because at too large a gain oscillations will occur. This sort of control is called “proportional control”.

The error can be reduced to zero by adding to the feedback a heater signal proportional to the integral of the out-of-balance signal. Thus the heater power continues to change until the temperature error is zero. This is called “integral” (U.S.: “reset”) control. Its disadvantages are that it introduces extra phase lags

into the control loop and gives the controller a greater tendency either to be unstable, or at least to overshoot.

The overall stability can be improved by adding in a third term to the feedback, proportional to the rate of change of temperature. This is “derivative” (or “rate”) control. Its action is analogous to a viscous damping term, e.g. in a SHO, with a force proportional to velocity. You’d better get its sign right or it’ll act as a source of instability! But too much differential control even of the right sign is also bad because it can amplify the effect of sudden temp changes leading to the heater switching hard on and hard off in a non-linear type of instability.

If the proportional control signal is represented by V , then the magnitude of the integral control can be written:

$$V_{\text{int}} = \frac{1}{\tau_{\text{int}}} \int V dt$$

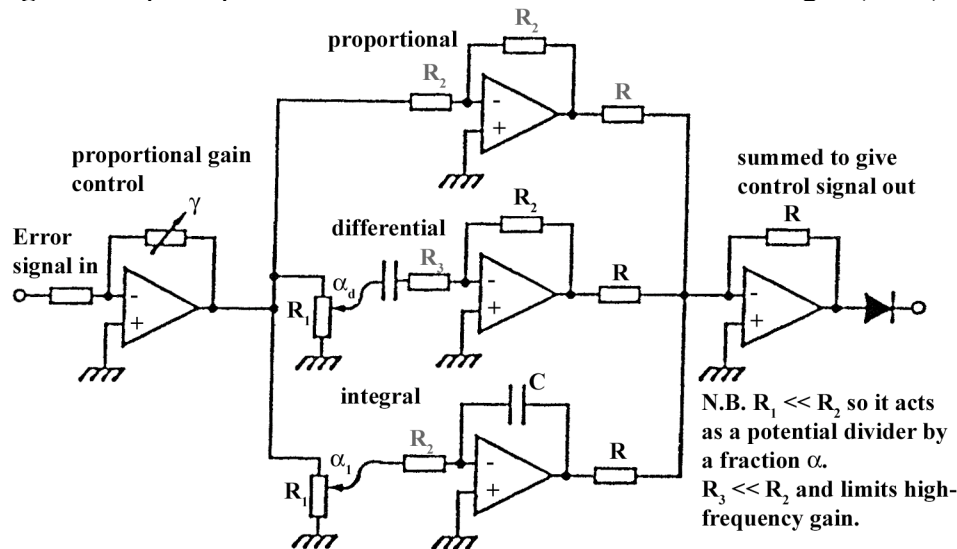
i.e. its size relative to the proportional signal can be represented by a time constant τ_{int} . Similarly for the size of the differential control relative to proportional control:

$$V_{\text{diff}} = \tau_{\text{diff}} \frac{dV}{dt}$$

But note that in one case large τ means a large signal, and in the other case a small signal.

Practical op-amp circuits for achieving these controls are shown in fig. 8.

Figure 8. Op-Amp cct for PID Control – better than that in Forgan (1974)



$$\tau_d = \alpha_d CR_2 : \tau_i = CR_2 / \alpha_i : \text{gain}(\omega) = \gamma (1 + j\omega\tau_d + 1 / j\omega\tau_i)$$

This may be written, as in Forgan, $\text{gain}(\omega) = \gamma (1 + j\omega\tau_3) (1 + j\omega\tau_5) / j\omega\tau_7$

d) Conditions for Stable Operation

In order to minimise phase shifts in the thermal circuit of fig.7, it is essential to have short thermometer and heater time-const. This is so that the heat gets straight to the block and any temp change of block is rapidly detected and corrected for (after all, the controller doesn't possess ESP: it can only respond to what the thermometer tells it!). What is perhaps less obvious is that the system also needs a long time constant too: e.g. the block or LIA time constant. The control works best when there is a large ratio between the different time constants in the system. In addition, the time constants of the proportional and integral control should be set somewhere near the middle of the other time constants. The maths to prove all this is not difficult, so if you wish to understand temp control better, consult E.M. Forgan, Cryogenics 14 pp207-14 (1974).

Typical Magnetic Field-Dependent Temperature Errors for Selected Cryogenic Temperature Sensors $\Delta T/T(\%)$ at β (magnetic field)

Sensor Type	T(K)	β (tesla units)				Comments
		2.5	8	14	19	
Carbon Glass Resistors	4.2	-0.5	-2.3	-4.9	-6.6	Negative $\Delta R/R$ when $T \geq 60K$. Good reproducibility. Reduced sensitivity above 100K. Errors negative below 20K.
	10	-0.2	-1.1	-2.6	-3.8	
	25	0.02	0.22	0.54	0.79	
	45	0.07	0.48	1.32	2.16	
	88	0.05	0.45	1.32	2.30	
	306	<0.01	0.22	0.62	1.14	
Platinum Resistors	20	20	100	250	-	Recommended for use when $T \geq 30K$
	40	0.5	3	6	8.84	
	87	0.04	0.4	1	1.74	
	300	<0.01	0.02	0.07	0.13	
Rhodium-Iron Resistors	4.2	11	40 (6 tesla)	-	-	Not recommended for use below 77K in magnetic fields.
	40	1.5	12	30	46.76	
	87	0.2	1.5	4	6	
	300	<0.01	0.1	0.4	-	
CS-401 (SrTiO ₃) Capacitors	2.2	<0.02	<0.02	0.02	-	Probably zero field-induced temperature error. Recommended for control purpose.
	4.2	<0.01	<0.01	0.01	-	
	88	<0.01	<0.01	<0.01	≤ 0.01	
	190	<0.01	<0.01	<0.01	≤ 0.01	
	300	-	-	-	-	
CS—501 Multilayer Ceramic Element Capacitors		$\Delta T/T(\%) < 0.15$ at 4.2K and 18.7 tesla. $\Delta T/T(\%) < 0.05$ at 77K and 305K and 18.7 tesla.				Recommended for control purposes. Monotonic in C vs. T to nearly room temperature.
Germanium Resistors	2.0	-8	-60	-	-	Not recommended except at low β owing to large, orientation-dependent temperature effect.
	4.2	-5 to -20	-30 to -55	-60 to -75	-	
	10	-4 to -15	-25 to -60	-60 to -75	-	
	20	-3 to -20	-15 to -35	-50 to -80	-	
Chromel-AuFe (0.07%) Thermocouples	10	3	20	30	-	Data taken with entire thermocouple in field, cold junction at 4.2K, errors in hot junction temperature.
	45	1	5	7	-	
	100	0.1	0.8	-	-	
Type E (Chromel-CuNi) Thermocouples	10	1	3	7	-	Useful when $T \geq 10K$. Refer to comments for Chromel-AuFe (0.07%).
	20	<1	2	4	-	
	45	<1	<1	2	-	

L.G. Rubin, B.L. Brands and H.H. Sample, some Practical Solutions to Measurement Problems Encountered at Low Temperatures and High Magnetic Fields. Advances in Cryogenic Engineering. Vol. 32, Plenum Press, New York (1986). p. 1221.

Typical Magnetic Field-Dependent Temperature Errors for Diode Temperature Sensors $\Delta T/T(\%)$ at β (magnetic field)

Sensor Type	T(K)	β (tesla units)					Comments
		1	2	3	4	5	
CaAlAs Diodes	4.2	2.9	3.8	3.7	2.8	1	Shown with junction perpendicular to applied field H. (When junction is parallel to H, induced errors are typically less than, or on the order of, those shown.)
	30	0.2	0.2	0.3	0.3	0.2	
	78	<0.1	<0.1	0.17	0.16	0.1	
	300	≤0.1	<0.1	<0.1	<0.1	<0.1	
Silicon Diodes	4.2	-200	-300	-350	-400	-500	Strongly orientation dependent.
	20	-10	-20	-25	-30	-40	Junction parallel to field.
	40	-4	-6	-8	-10	-12	
	60	-0.5	-1	-2	-3	-3.5	
	80	<0.1	<0.5	-0.8	-1.1	-1.5	
	300	<-0.1	<-0.1	<-0.1	<-0.1	<-0.1	
Silicon Diodes	4.2	-8	-9	-11	-15	-20	Junction perpendicular to field.
	20	-4	-5	-5	-0.5	10	
	40	-1.5	-3	-4	-5	-5.5	
	60	-0.5	-0.7	-0.8	-1	-1.1	
	80	-0.1	-0.3	-0.5	-0.6	-0.7	
	300	<0.1	0.2	0.5	0.6	0.6	

Figure 9. Illustration of the similarity of temperature versus resistance curve characteristics within each Germanium resistor model type. Shown are the plotted resistance-temperature curves for six Model GR-200A-1000 sensors.

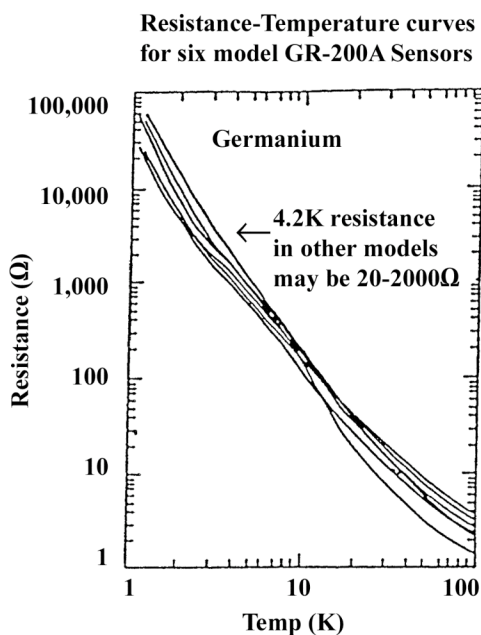


Figure 10. Carbon / Glass resistance.

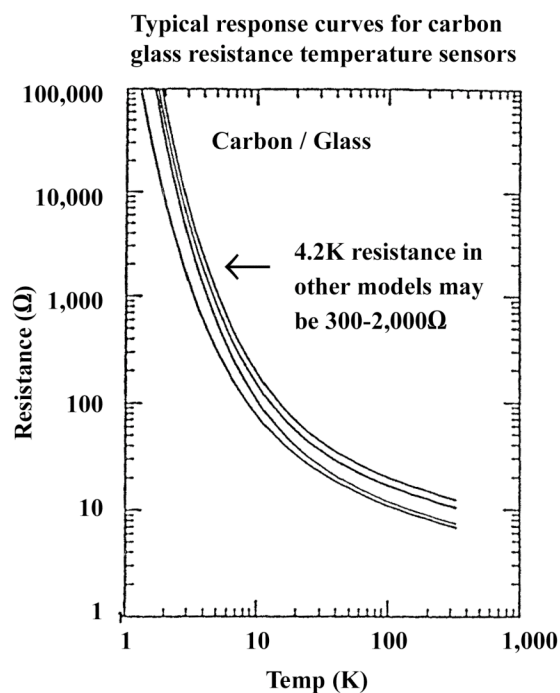


Figure 11. Resistance versus temperature (typical curve) for RF-800-4 (27 ohm) Rhodium Iron resistors.

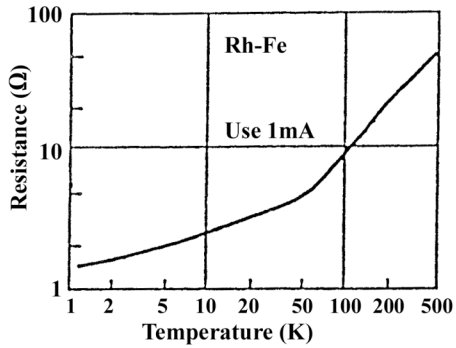


Figure 12. Typical temperature response curve for CS-501.

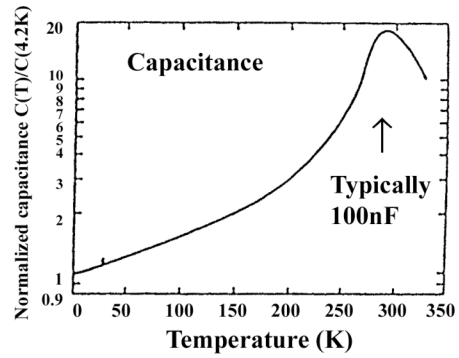


Figure 13.

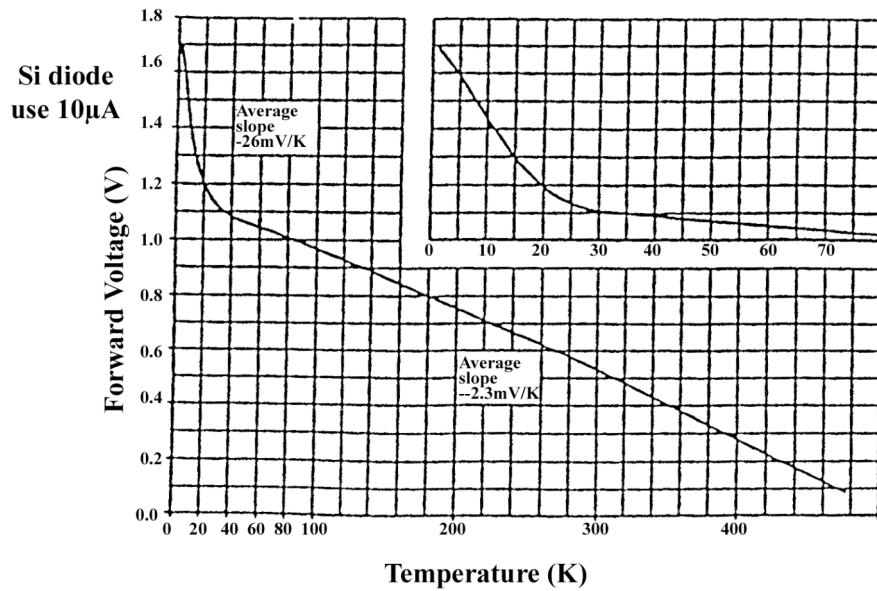


Figure 14.

